Analysis of Human Oto-Acoustic Emissions

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Transient-evoked oto-acoustic emissions (TEOAEs, also called click-evoked or delayed evoked) are useful because they yield non-invasively information about the cochlear organs, even if it is impossible to communicate with the subject (e.g., in the case of babies).

If the outer hair-cells (OHCs) are damaged, then the TEOAEs in the corresponding frequency region are weak or absent.

In the present analysis of TEOAE experiments documented in the literature, the time-dependence of the instantaneous frequency of the emissions is shown to be consistent with a cochlear model [Frosch (2009, 2010)] involving two cochlear resonators, namely the internal organ-of-Corti resonator (IOCR, see below) and the basilar-membrane resonator (BMR).
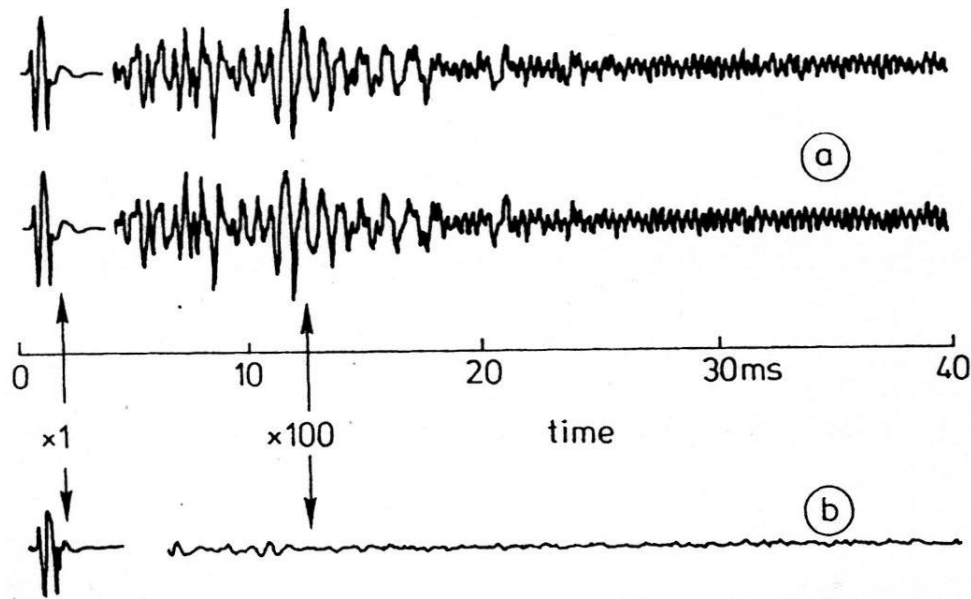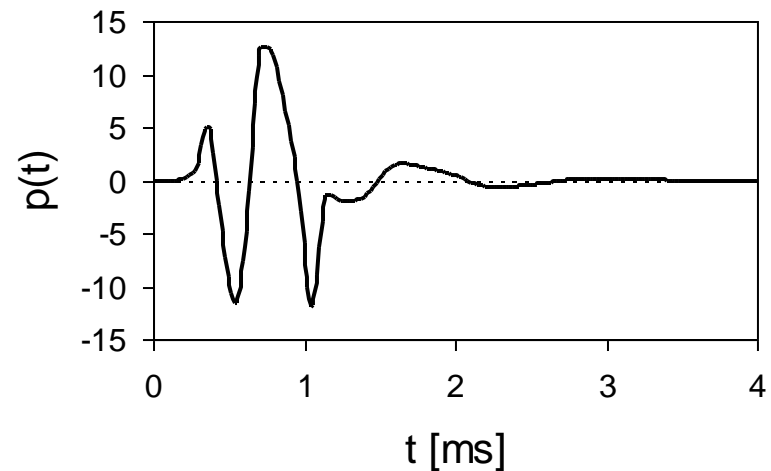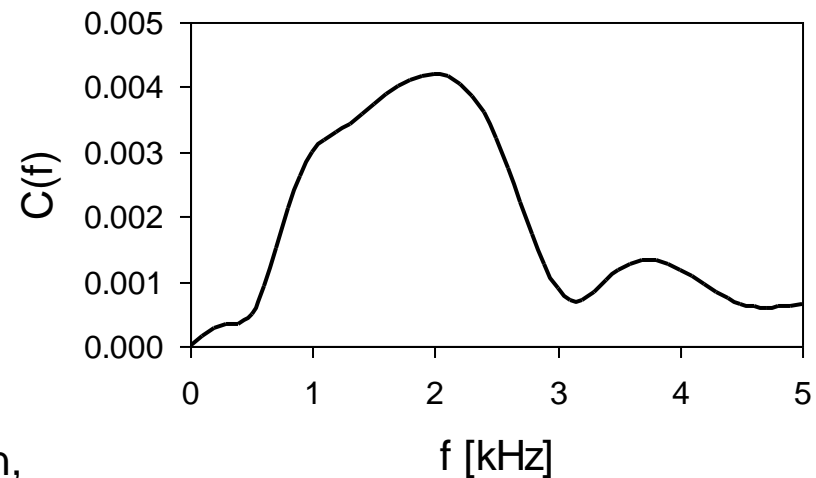
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Magnified representation of click.

Fig. 16.13 of Zwicker and Fastl (1999).
Sound source and microphone in closed ear canal.
a) healthy inner ear of baby;
b) human subject suffering from hearing loss.

The click (0 to 2 ms) and the TEOAE (4 to 17 ms)
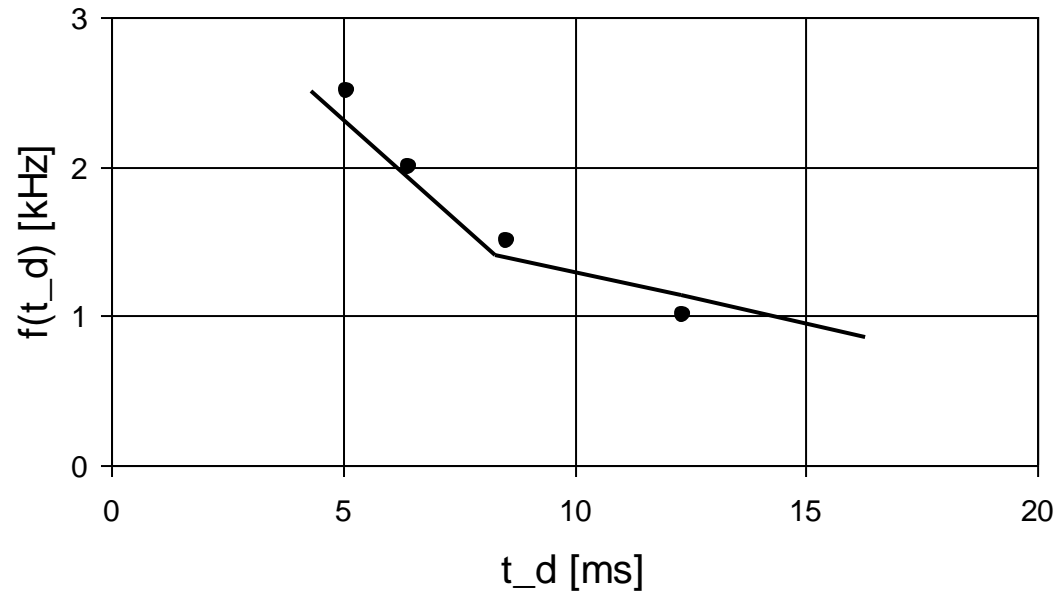both contain frequencies from ~1 to ~2.5 kHz.

At t > 14 ms, there is a „spontaneous" 3-kHz emission,
(see later), triggered by the click.

Frequency spectrum of click.

2

t_d = delay after click maximum = t - 0.7ms ;
f(t_d) = instantaneous frequency of TEOAE;
solid line: experimental frequencies, taken from Fig. 16.13a of Zwicker and Fastl;
filled circles: theoretical frequencies, based on surface-wave formulae (see below).

A stationary pure tone, i.e., a sinusoidal tone of time-independent frequency and amplitude,

generates a travelling wave in the cochlear channel.

That wave is similar to a mass-loaded surface wave on a lake

(e.g., a lake covered by floating pieces of ice).

The wave in the cochlea, however, is not gravity-driven, but spring-driven;

springs = fibres of the basilar membrane; mass load = basilar membrane + attached cells.

The wave in the cochlea is similar to that in a box model.
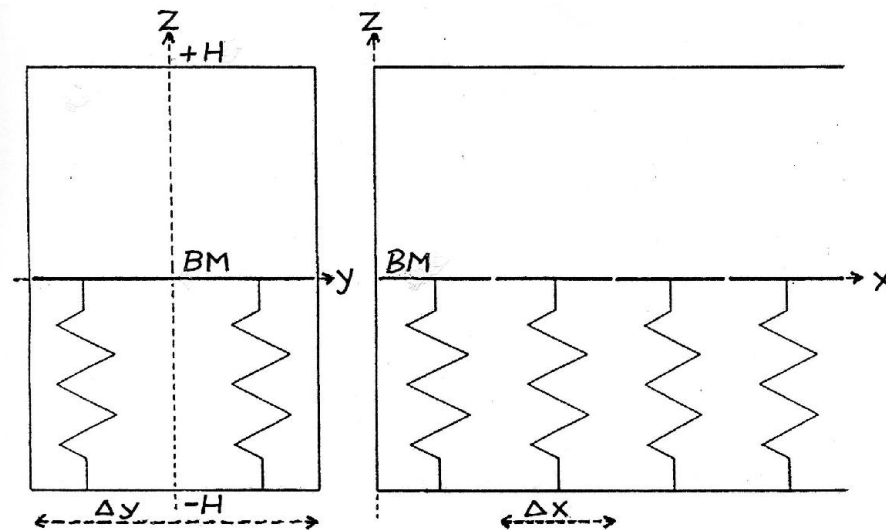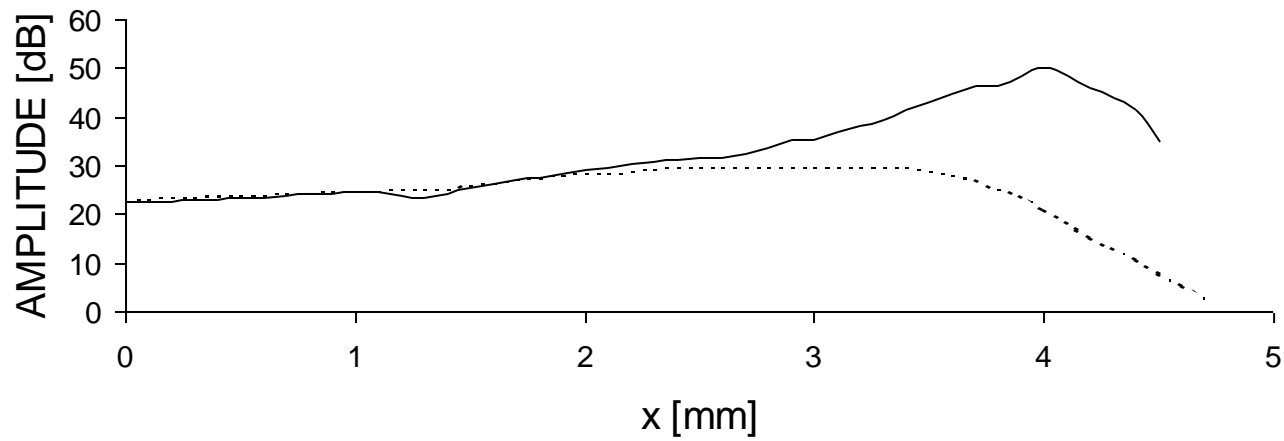
Experimental BM oscillation velocity (in dB, re stapes) versus distance x from stapes; guinea-pig; f = 17 kHz; from E. de Boer (2006);

dotted curve: post-mortem (any SPL) or healthy cochlea (SPL = 100 dB);

solid curve: healthy cochlea (SPL = 20 dB).

Passive-peak map = PP map = peak of dotted curve; thus, for guinea pig:

x_{PP}(17 kHz) = 2.8 mm;

(Low-level) active-peak map = AP map = peak of solid curve; for guinea pig:

x_{AP}(17 kHz) = 4.0 mm = „characteristic place".

During a stationary pure tone, wave energy is fed into the cochlea at the stapes.

The BM oscillation velocity increases with x because the group velocity decreases.

Box-model formula for c_{group} (for x-independent properties, without friction, short-wave approximation):
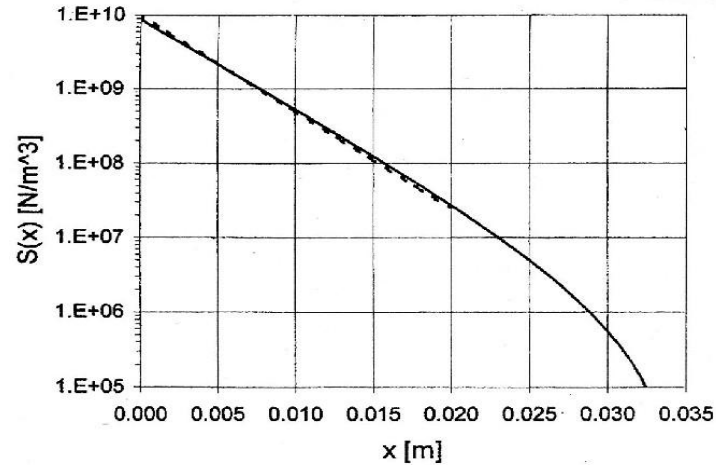
$$c_{\text{group}} = \frac{S}{4\rho\omega} \cdot \left(1 - \frac{M\omega^2}{S}\right)^2 ;$$

$S\,[\text{N}/\text{m}^3] = \text{Stiffness of BM};$

$M\,[\text{kg}/\text{m}^2] = \text{Surface mass density of BM};$

$\omega\,[\text{s}^{-1}] = 2\pi f = \text{angular frequency of tone};$

$\rho \approx 1000\,\text{kg}/\text{m}^3 = \text{liquid density}.$

The BM surface mass density is approximately x-independent; humans: $M=0.1\text{kg}/\text{m}^2$.

The BM stiffness $S(x)$ decreases strongly from $x=0$ to $x=L$.; see diagram above (for homo).

A third cochlear map: the basilar-membrane resonator map = BMR map;　　$f_{\text{BMR}}(x) = \frac{1}{2\pi} \cdot \sqrt{\frac{S(x)}{M}}$.

$f_{\text{BMR}}(x)$ is the resonance frequency without liquid; at $f_{\text{BMR}}(x)$ the group velocity vanishes.

Without friction, the BM oscillation velocity at $x_{\text{BMR}}(f)$ would be infinite; with frictional heat generation,

there is a maximum of the BM oscillation velocity at $x_{\text{PP}}(f)$, and the wave is weak at $x_{\text{BMR}}(f)$.
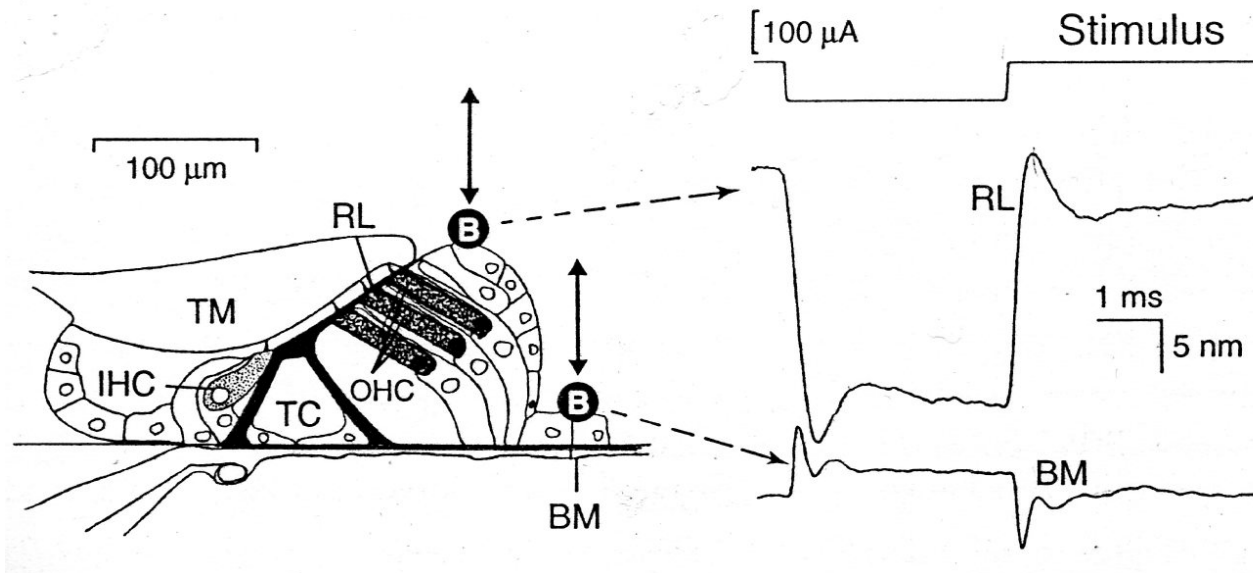
Excised-cochlea laser-interferometer experiment of Mammano and Ashmore, 1993; guinea-pig; x=11mm; B: reflecting beads; RL: reticular lamina.

The strongly damped BM oscillations imply $f_{BMR}(x) / K$ = 2.3 kHz ;
the strongly damped RL oscillations imply $f_{IOCR}(x) / K$ =1.0 kHz;
K = correction factor due to evanescent waves (about 1.4, see page 11).

A fourth cochlear map: the internal organ-of-Corti resonator map = IOCR map;
springs = OHCs and other nearby structures; angle formed by BM and RL varies periodically.

The IOCR oscillations cause the OHCs to feed mechanical energy into the TW and thus to give rise to the active peak; source of energy: electric current through OHCs, modulated by the stereocilia of the OHCs.
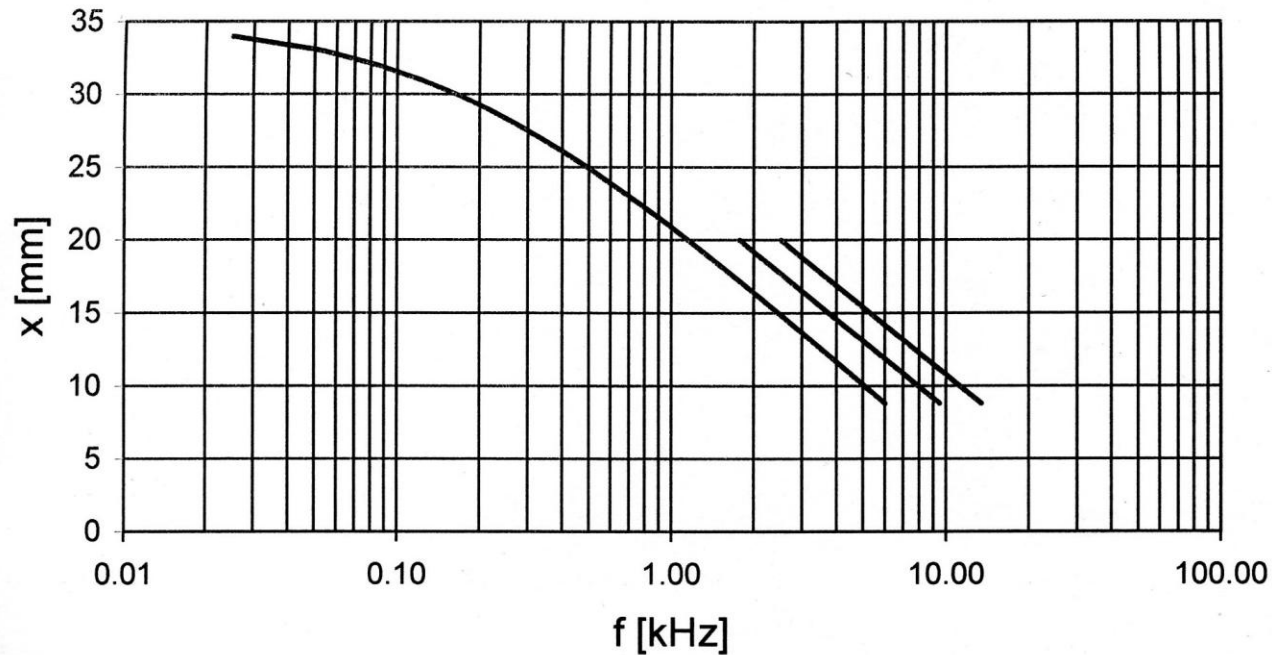
Preliminary cochlear maps for homo;

lowest curve (0.025-6 kHz): human cochlear map according to Greenwood (1990), conjectured to be close to the passive-peak map and, at f > 1 kHz, to the internal-organ-of-Corti-resonator map;

highest line (2.5-13 kHz): basilar-membrane-resonator map according to de Boer (1996); one-octave difference between PP and BMR maps agrees with data from several mammals;

middle line (1.8-9.5 kHz): active-peak map, 0.5 octave below BMR map, inferred from maps of chinchilla, guinea-pig, and gerbil.

Interpretation of Fig. 16.13b of Zwicker and Fastl:
outer hair cells in basal half of cochlear channel were damaged, travelling waves generated by the click were passive, were not significantly reflected, and were extinguished by friction after having passed their passive-peak place.

Interpretation of Fig. 16.13a:
Any strong component of the click frequency spectrum caused the outer hair cells (OHCs) in the corresponding internal-organ-of-Corti resonance region to feed energy into the travelling wave, and also to generate a backward travelling wave, which carried some of the wave energy generated by the OHCs back towards the stapes.

Predicted delay of the TEOAEs:

$$t_d(f) = 2\,\tau_{\mathrm{SW}} + \tau_{\mathrm{rise}} \; ,$$

where $\tau_{\mathrm{SW}} =$ surface-wave group travel time, from stapes to $x = x_{\mathrm{IOCR}}$, according to box-model

short-wave formula, $\tau_{\mathrm{SW}} = \dfrac{4\rho \cdot \omega}{\alpha \cdot S_0} \cdot \left(\dfrac{1}{e^{-\alpha \cdot x} - \eta'} - \dfrac{1}{1 - \eta'} \right);$ $\rho =$ density of water; $\omega = 2\pi \cdot f;$

BM stiffness $S = S_0 \cdot e^{-\alpha \cdot x};$ $\eta' = \dfrac{M \cdot \omega^2}{S_0};$ $M =$ BM surface-mass density ;

$\tau_{\mathrm{rise}} =$ rise time of IOCR oscillatio n; $\tau_{\mathrm{rise}} = \dfrac{Q}{\pi} \cdot \dfrac{1}{f} \approx \dfrac{1.3}{f};$ $Q \approx 4.$

That theoretical function $t_d(f)$ agrees fairly well with experiment; see page 3.

The just presented box-model surface-wave formulae predict also, e.g., the experimental
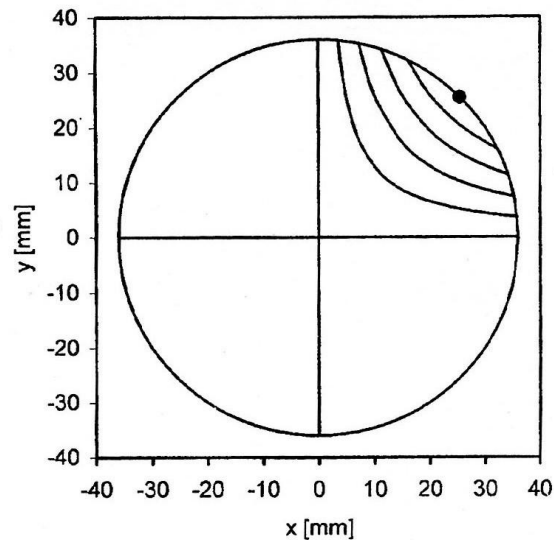TEOAE delays shown in Fig. 3.20 of Zwicker and Fastl (1999):
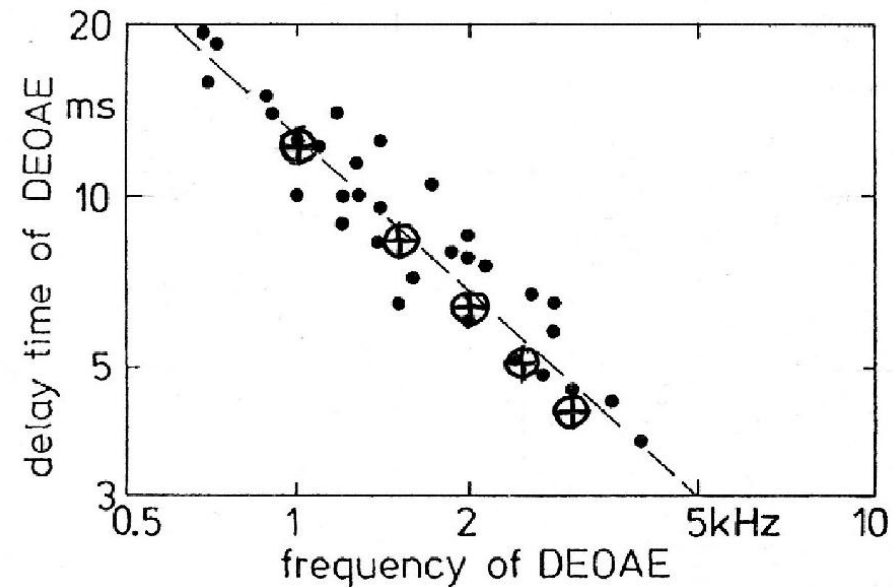
Filled circles: measured delays; dashed line: fit of Zwicker and Fastl, $t_d = 1/\,df(0.4$ bark$)$;
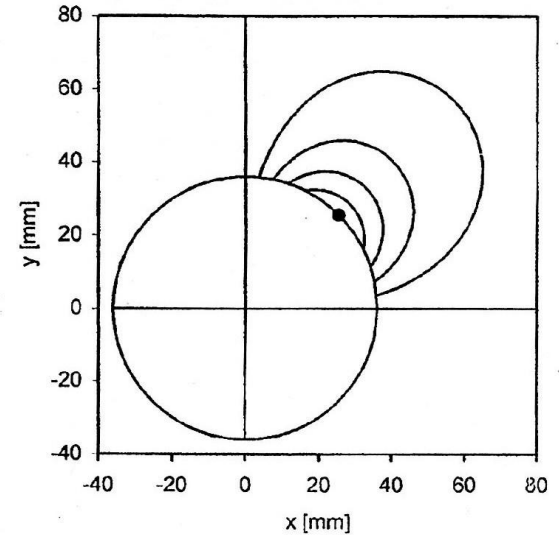large hand-drawn symbols: box-model surface-wave formulae, see page 9.

The „spontaneous" 3-kHz emission in Fig. 16.13a of Zwicker and Fastl may be caused by feedback-generated BMR oscillations involving evanescent liquid sound-pressure waves. These oscillations are similar to those of underwater resonators, e.g., tapped wineglasses. A tapped submerged wineglass yields an unmusical click if the water is disturbed, e.g. by a beam of water from above; a tone of well-defined frequency, ringing for about one second, results only if the water is quiet.
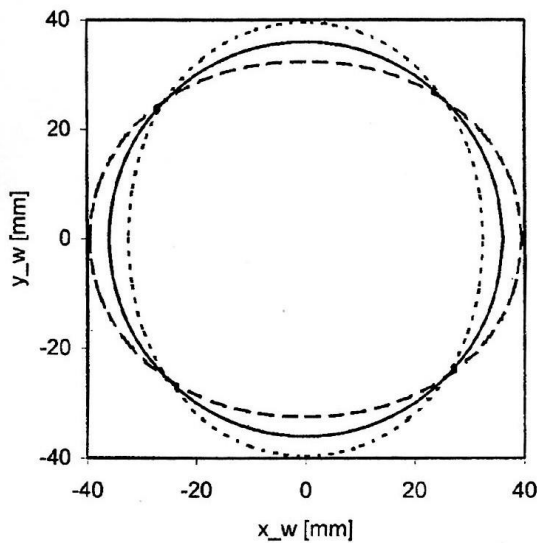
Oscillation pattern.   Streamlines inside.   Streamlines outside.

The tone of the submerged glass is lower than that of the empty glass in air by ~1.2 octave. A submerged tuning fork, however, yields ~415 Hz, lower than 440 Hz by a semitone only. The resonance frequency of a submerged BMR is conjectured to be lower than the (without-liquid) BMR frequency by ~0.5 octave.

Conjectured place of the submerged BMR generating the 3-kHz emission: $x = 16.6$ mm.
Resonance frequency of local BMR without liquid (page 8): f (BMR) = 4.2 kHz.
Resonance frequency of local BMR with evanescent waves: 3.0 kHz.
Delay time t_d $(= t - 0.7\text{ms})$ of the start of the full-amplitude 3-kHz oscillation in Fig. 16.13a (page 2):

$$t_d = \tau_{SW}(f_{max}, x) + \Delta t_1 + \Delta t_2 + \tau_{SW}(3\,\text{kHz}, x) = 14 \text{ ms, in agreement with page 2;}$$

$f_{max} = 2.5\,\text{kHz} = \text{highest strong component of click spectrum (page 2);}$

$\Delta t_1 \approx 1\text{ms} = \text{time after passage of } f_{max}\text{-component required until liquid is sufficient ly quiet;}$

$\Delta t_2 \approx 1\text{ms} = \text{result of attempt to estimate rise time of 3-kHz oscillatio n;}$

$\tau_{SW} = \text{wave-group travel time, see page 9.}$

A detailed description of this analysis is presented in Chapter 44 of Frosch (2010).

Conclusions:

1) The measured dependence of the click-evoked oto-acoustic-emission (OAE) delay on instantaneous frequency is consistent with the hypothesis that these emissions are generated by the outer hair-cells (OHCs) which feed wave energy into the forward travelling wave.

2) The place of these OHCs is the place of that internal organ-of Corti resonator (IOCR) having a resonant frequency equal to the considered click-component frequency; above ~1 kHz, that place is basal of the characteristic place (=low-sound-level active-peak place) by ~0.5 octave distance, and basal of the corresponding basilar-membrane resonator (BMR) by ~1 octave distance.

3) The „spontaneous" OAEs are hypothesized to be due to BMR oscillations which involve evanescent (standing) liquid-sound-pressure waves and so have a frequency ~0.5 octave below the local without-liquid BMR resonance frequency; these oscillations are thought to be generated by feedback from a local IOCR having a resonance frequency region ranging up to exceptionally high frequencies.

References:

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